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European Journal of Mechanics B/Fluids 23 (2004) 107–114

A high-amplitude 2T mode of vortex-induced vibration for a light body in XY motion

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Received 15 September 2003; accepted 24 September 2003

Abstract

Although there are a great many papers dedicated to the problem of a cylinder vibrating transverse to a fluid flow (Y-motion), there are almost no papers studying the more practical case of vortex-induced vibration in two degrees of freedom (XY-motion) where the mass and natural frequencies are precisely the same in both X and Y directions. We have designed the present pendulum apparatus to achieve both of these criteria. Even down to the low mass ratios, where $m^* = 6$, it is remarkable that the freedom to oscillate in-line with the flow, affects the transverse vibration surprisingly little. There is, however, a dramatic change in the fluid–structure interactions when mass ratios are reduced below $m^* = 6$. A new amplitude response branch with significant streamwise motion appears, in what we call the "super-upper" branch, yielding massive amplitudes of 3 diameters peak-to-peak ($A_Y^* \sim 1.5$). We discover a corresponding periodic vortex wake mode, comprising a triplet of vortices being formed in each half cycle, in what we define as a "2T" mode. The extensive studies of VIV for Y-only body motions, built up over the last 35 years, remain of strong relevance, for $m^* > 6$. It is only for "light" bodies, $m^* < 6$, that one observes a rather marked departure from previous results.

Keywords: Vortex-induced vibration; Vortex; Vibration; Resonance; Response

1. Introduction

Vortex-induced vibration of cylinders free to respond transverse to the fluid flow (Y-motion) has been well studied, following some early classical work of Feng [1] and others. Several reviews discuss this problem (see Sarpkaya [2], Bearman [3] and Williamson [4], for example). The work of Feng at high mass ratios, $m^* = 320$, demonstrates that the resonance of a body, when the oscillation frequency (f) coincides with the vortex formation frequency (f_V), will occur over a regime of normalised velocity U^* (where $U^* = U/f_N D,\ U =$ free-stream velocity, $f_N =$ natural frequency; $D =$ diameter) such that $U^* \sim 5-8$. Two response amplitude branches are found, which are shown by Brika and Laneville [5] and by Govardhan and Williamson [6] to be due to two modes of vortex formation, as follows. For the "Initial" branch of response, the vortex wake comprises a "2S" mode, representing *2 single vortices* formed per cycle. The "lower" branch comprises the "2P" mode, whereby *2 vortex pairs* are formed per cycle (as originally defined in Williamson and Roshko [7] from their forced vibration study). However, at low mass and damping (m^* typically of the order 5–10), and higher amplitudes of response, three response branches are found to exist (Khalak and Williamson [8,9], Govardhan and Williamson [6]), namely the Initial branch (with a 2S vortex wake mode), the Lower branch (with a 2P mode), and a further distinctly higher-amplitude mode appearing between these two other branches, namely the Upper branch (also with a 2P mode of vortex formation).

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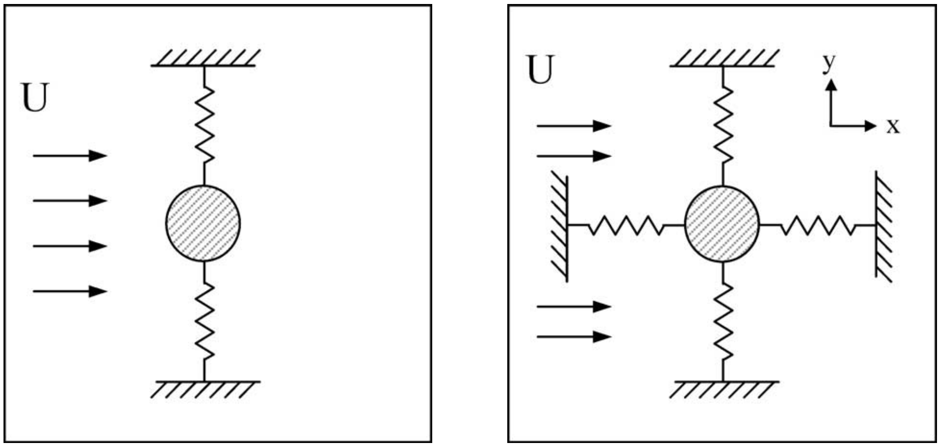
0997-7546/$ – see front matter
doi:10.1016/j.euromechflu.2003.09.008



(a) Y-only motion (b) XY motion

Fig. 1. Definitions of the XY-motion experiments. In (a) and (b), we show schematic diagrams of Y-only and XY motion experimental arrangements.

Despite the large number of papers dedicated to the problem of a cylinder vibrating transverse to a fluid flow (Y-motion, as in Fig. 1(a)), there are very few papers which also allow the body to vibrate in-line with the flow (in XY-motion, as in Fig. 1(b)). One of the principal questions which may be posed is: *How does the freedom to vibrate in-line with the flow influence the dynamics of the fluid and the structure*? In most of the past experimental work with XY vibrations (Moe and Wu [10], Sarpkaya [11]), the mass ratios and natural frequencies have, in general, been chosen to have different values, except for one data set for the same frequency (but different mass) in Sarpkaya. Under their chosen special conditions, these studies find a broad regime of synchronisation, similar to Y-only studies, but no evidence of the different response branches. Sarpkaya concludes from his work that bodies in XY-motion do not lead to surprising changes in the expected maximum resonant amplitudes as compared to bodies in Y motion. A different approach was adopted recently by Jeon and Gharib [12], who forced a cylinder to move in the X and Y directions, in a fluid flow, under the prescribed motions given by: $x(t) = A_X \sin(2\omega t + \theta),\ y(t) = A_Y \sin(\omega t)$. Specific phase angles: $\theta = 0°$and $-45°$, were chosen, since they stated that "nature prefers a figure-eight type motion". One of the most interesting results from the study of Jeon and Gharib appears to be the fact that even small amounts of streamwise motion ($A_X/A_Y = 20\%$) can inhibit the formation of the 2P mode of vortex formation. It might be noted that, in the free vibration study of Jauvtis and Williamson [13], they find body motions which can be quite different from a figure-of-eight motion. Clearly, the selection of amplitudes and phases will influence the resulting conclusions.

It is significant to note that full-scale piles in an ocean current (Wooton et al. [14]), and similar cantilever models in the laboratory (King [15]), have been found to vibrate in-line with the flow with peak amplitudes of the cantilever tip ($A_X^* = A_X/D \sim 0.15$). As noted by Bearman [3], and by Naudascher [16], who provides an excellent review of in-line vibrations, oscillations ensue if the velocity is close to $U^* \sim 1/2S$. King showed a classical vortex street (antisymmetric) pattern, although these investigators also discovered a second mode where the wake formed symmetric vortex pairs close to the body. However, in the present work, we are interested primarily in the modes which exhibit large transverse vibrations.

In most practical cases, cylindrical structures (such as riser tubes or heat exchangers, to name but two examples) have *the same mass ratio and the same natural frequency* in both the streamwise (X) and transverse (Y) directions. Two recent arrangements which ensure such conditions, are the air bearing platform of Don Rockwell's group at Lehigh University, and a pendulum set-up at Cornell (Jauvtis and Williamson [13,17]). Both of these research groups demonstrate a set of response branches, in contrast to previous XY experiments. Even down to the low mass ratios, where $m^* = 6^*$, it is remarkable that the freedom to oscillate in-line with the flow, hardly affects the response branches, the forces, and the vortex wake modes. These results are significant, because they indicate that the extensive understanding of vortex-induced vibration for Y-only body motions, built up over the last 35 years, remain of strong relevance to the case of two degrees of freedom.

Further exploration to much lighter structures, in this paper, reveals the existence of a new and remarkably high-amplitude vibration mode, as one reduces the mass ratio to around, $m^* = 4$. This response mode has a peak-to-peak amplitude of 3 diameters ($A_Y^* = 1.5$), and is caused by a new mode of vortex dynamics, that does not occur for strictly Y-only motion; namely a "2T" mode, comprising a *triplet of vortices* that forms in each half cycle of transverse body motion. Not only is the amplitude significantly larger than found hitherto for the 2P mode (typically $A_Y^* = A_Y/D = 1$) but also the 2T mode is found to be more stable and periodic.

2. Experimental details

We have constructed a hydroelastic apparatus, for particular application to very low mass and damping conditions, which operates in conjunction with the Cornell-ONR Water Channel. One may refer to Khalak and Williamson [8] for the details concerning this water channel facility, whose Lucite test section is 38.1 cm wide, with a water depth of 46 cm, and with a length of 250 cm. A horizontal plate over the water channel is suspended by four cables from the roof of the laboratory, and this plate acts as a pendulum, below which is mounted a vertical cylinder that reaches down into the fluid flow of the water channel. The cylinder is thus able to move in-line and transverse to the free stream, and has the same natural frequency (typically $f_N = 0.4$ Hz.) and oscillating mass (m^* from 2.0 to 25.0) in these two directions, which was an essential design requirement. Cylinders of submerged length = 38.1 cm, and diameter = 3.81 cm or 5.08 cm, were used, in the same configuration as Khalak and Williamson [8]. Very low values of the mass-damping parameters were used; $(m^* + C_A)\zeta = 0.001$ to 0.1 (where C_A = Ideal added mass coefficient = 1.0, and ζ is the structural damping coefficient). Reynolds numbers ranged from 1,000 to 15,000. Displacement was measured using magneto-strictive (non-contact) instrumentation. Digital Particle Image Velocimetry (DPIV) was used to determine the vorticity in a plane midway down the submerged cylinder length, and the implementation of this technique is described in detail in Govardhan and Williamson [6]. In colour contour plots of vorticity in this paper, the blue colour represents clockwise vorticity, while the red is for anticlockwise vorticity. Forces on the body are measured, simultaneously with the measurement of displacements and vorticity field, and we describe the force measurement using LVDTs in more detail in Khalak and Williamson [8,18]. The coordinate system is defined such that the origin is where the cylinder axis intersects the free surface; x is the downstream axis, y is the transverse axis, and z is the downwards vertical axis of the cylinder. In this paper, A_X^* and A_Y^* refer to streamwise and transverse amplitudes, normalised by diameter, D. Normalised frequency, $f^* = f/f_N$, where f = actual body oscillation frequency, and f_N = natural frequency of the body in the fluid medium (water). Strouhal number is defined as $S = f_V D/U$, where f_V = vortex shedding frequency of the non-oscillating body.

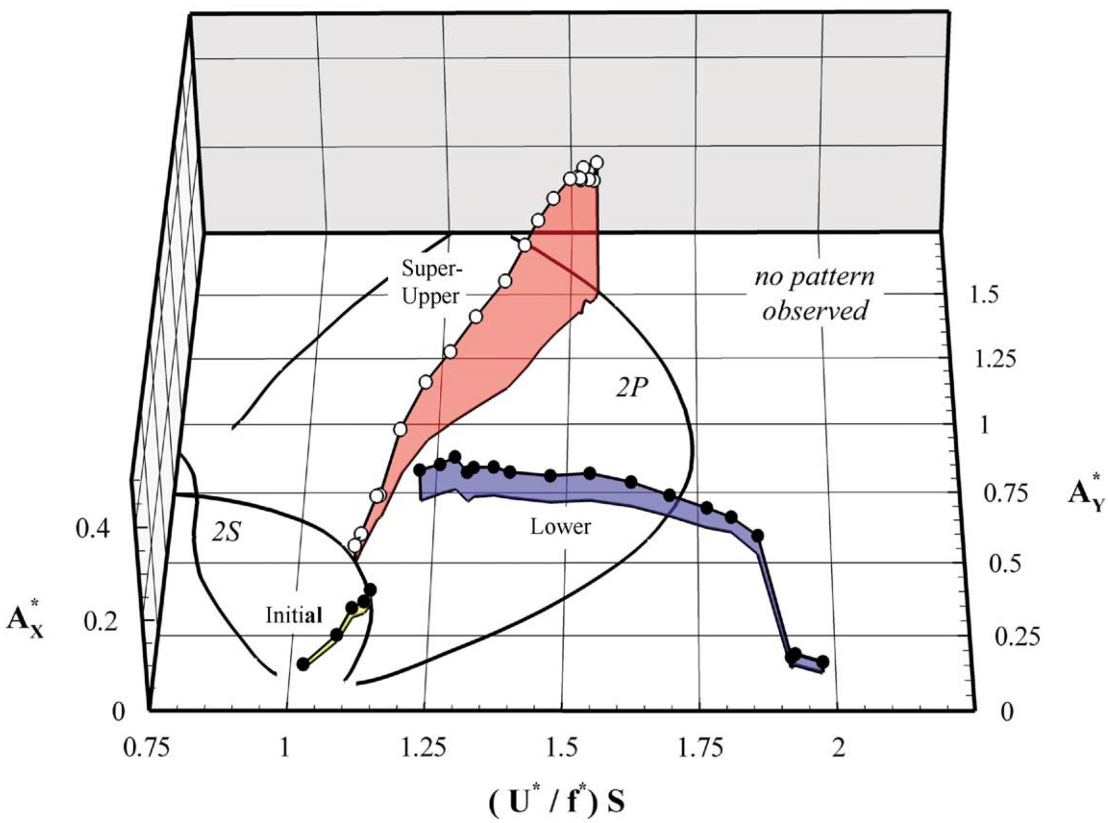


Fig. 2. Three-dimensional amplitude response plotted versus true reduced velocity, $(U^*/f^*)S$. The streamwise amplitude, A_X^* is the vertical axis, and we superpose the Williamson and Roshko map of flow regimes in the horizontal plane where we have Y-only motion. The super-upper branch, in red, flies well above the map of regimes, yet interestingly starts and terminates above the 2P mode boundaries. [$m^* = 2.6$, $(m^* + C_A)\zeta = 0.013$].

3. High-amplitude 2T vortex wake mode for small mass ratios ($m^* < 6$)

If we reduce the mass ratio to $m^* = 2.6$, the first striking feature to appear in the amplitude response plot of Fig. 2, is the remarkably high amplitude of response, up to 3 diameters peak-to-peak, or $A_Y^* = 1.5$. As we conducted the experiment, it was with some surprise that, as the velocity was gradually increased, the amplitude kept increasing far beyond any value we had hitherto experienced in our Y-only experiments (where typical peak amplitudes are of the order $A_Y^* \sim 1$; see Govardhan and Williamson [6]). We define this high-amplitude regime as the "super upper" branch of response, as distinct from the "upper" branch of Y-only studies.

For given mass and damping, the response amplitude will be a function of its oscillation frequency (f) relative to the frequency of vortex shedding in the absence of vibration (f_V). We therefore employ, in Fig. 2, a normalised velocity $(U^*/f^*)S$, which is equal to (f_V/f). In the present case, using the plane $\{(U^*/f^*)S,\ A_Y^*\}$, we can also overlay the results onto the Williamson and Roshko [7] map of vortex mode regimes found for Y-only forced vibration. We have also generalised the response plot by including the streamwise amplitude (A_X^*) as a third axis in a 3D plot, where each response branch height is represented by a "wall" of a specific colour. This is a useful presentation as it vividly exhibits the vibrations in the two principal directions. One can note that the horizontal plane represents the Y-only Williamson–Roshko map of regimes, and that other "solution surfaces" might exist above this plane, in the case of XY motion, which would be chosen by the system, yielding net positive energy transfer from fluid to body motion. Our present super-upper branch represents just one possible line in such a surface.

Within the super-upper branch, the in-line vibration amplitude, $A_X^* \sim 0.3$, is significantly higher than the amplitudes of the in-line modes that occur at low $U^* \sim 2.5$ (described in Jauvtis and Williamson [13], corresponding to the cantilever modes of Wooton et al. [14] and King [15].) It is quite interesting that the initial branch, with a 2S mode of vortex formation, terminates at the boundary of the 2S mode regime in the Williamson and Roshko map. At this point, there is a clear jump from the initial branch to what we define as the super-upper branch. The lower branch exhibits the same character as found for Y-only dynamics: a reasonably constant level of amplitude and frequency, from the start of the branch at $U^* \sim 8$ until this synchronisation ends at $U^* \sim 12$. There is a small but discernible level of streamwise vibration ($A_X^* \sim 0.05$).

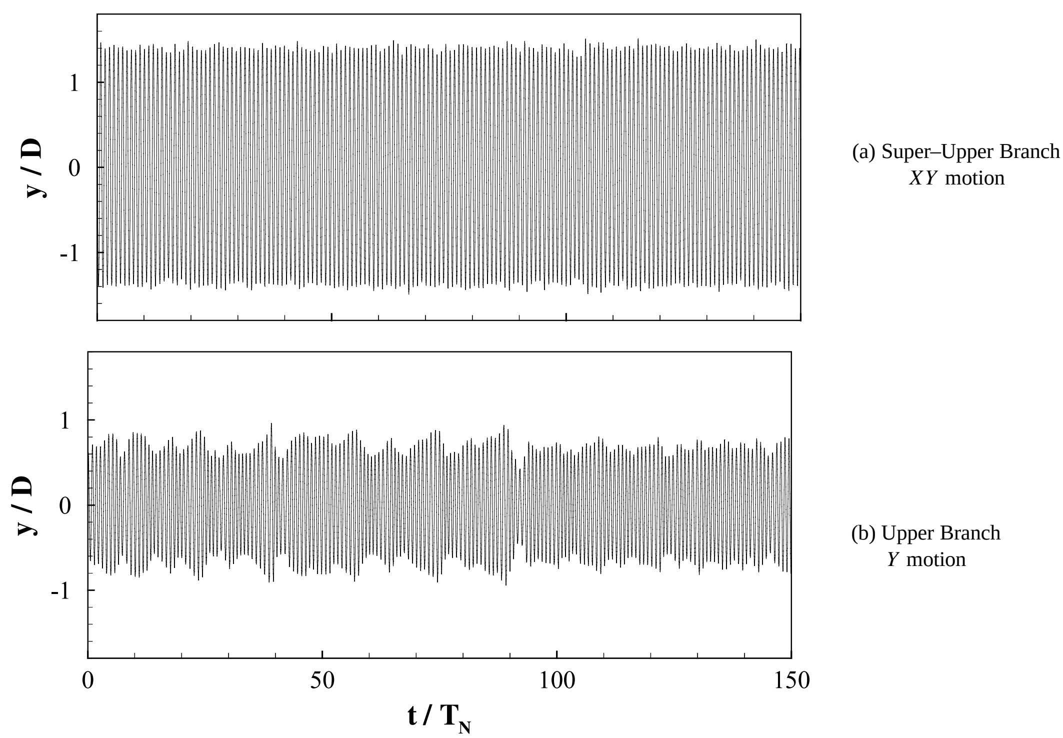


Fig. 3. Time traces of displacement $y(t)$, showing in (a) the remarkable periodicity for the vibrations in the super-upper branch, despite the huge amplitudes. These oscillations are compared with the more intermittent vibrations of the upper branch of Y-only motion in (b). For both cases, $(m^* + C_A)\zeta = 0.013$. For (a), $U^* = 6.65$; for (b), $U^* = 8.35$.

Although the super-upper branch will be shown in this paper to correspond with a vortex mode different from the 2P mode, it is very interesting that it starts at a low-amplitude 2P boundary, and also terminates at a (high-amplitude) 2P mode boundary in the Williamson–Roshko map (after which the vortex formation desynchronises from the body motion in this map). The significance of the super-upper branch for XY vibrations terminating at the 2P boundary (relevant to Y-only motion), is not clear, and might represent a coincidence. Another surprising feature of such a presentation is the remarkably large region of overlap between the super-upper and lower branches, extending over a range of velocities $(U^*/f^*)S = 1.20-1.55$. In this range, if one chooses a particular $(U^*/f^*)S$, the system can operate at either of two amplitude solutions, for the given mass and damping values.

Implementing the Hilbert Transform on the time traces of force and displacement, Khalak and Williamson [9] were able to show that the transition between the upper and lower branches (for Y-only motion) involved an intermittent switching between these branches of comparable amplitude. An example time-trace of displacement $y(t)$ for Y-only motion close to the upper $\Leftrightarrow$ lower branch transition is shown in Fig. 3(b), and demonstrates the intermittent nature of the amplitude envelope, in marked contrast with the case for XY dynamics in the super-upper branch, shown in Fig. 3(a), where vibrations are remarkably periodic. In the present case, if the flow was to switch mode, it would have to do so between markedly dissimilar amplitudes of $A_Y^* = 1.5$ and $A_Y^* = 0.7$, which is a jump in amplitude far larger than the jump associated with mode switching in Y-only motion. Instead of this scenario, we find that the super-upper $\Leftrightarrow$ lower transition is *hysteretic*. (This is described further in Jauvtis and Williamson [19].) It is therefore consistent that the mode remains comfortably stable in the super-upper branch, and the displacement time trace $y(t)$ is remarkably stable and periodic, despite the very high amplitudes of the body vibration.

The appearance of a new type of response branch, for sufficiently low mass structures in XY vibration, would suggest that there is a corresponding vortex formation mode, quite distinct from the known modes in vortex-induced vibration. This is indeed the case, as shown in Fig. 4, where we find a new mode of vortex formation comprising three vortices being formed per half cycle of body motion. We define this the "2T" mode, comprising 2 *triplets* of vortices per cycle (in accordance with the terminology introduced in Williamson and Roshko [7]). In this mode, we may observe vortices 1 and 2 in Fig. 4(a), comprising a counter-rotating vortex pair, which can be compared directly to the vortex pair of the 2P mode found in Govardhan and Williamson [6].

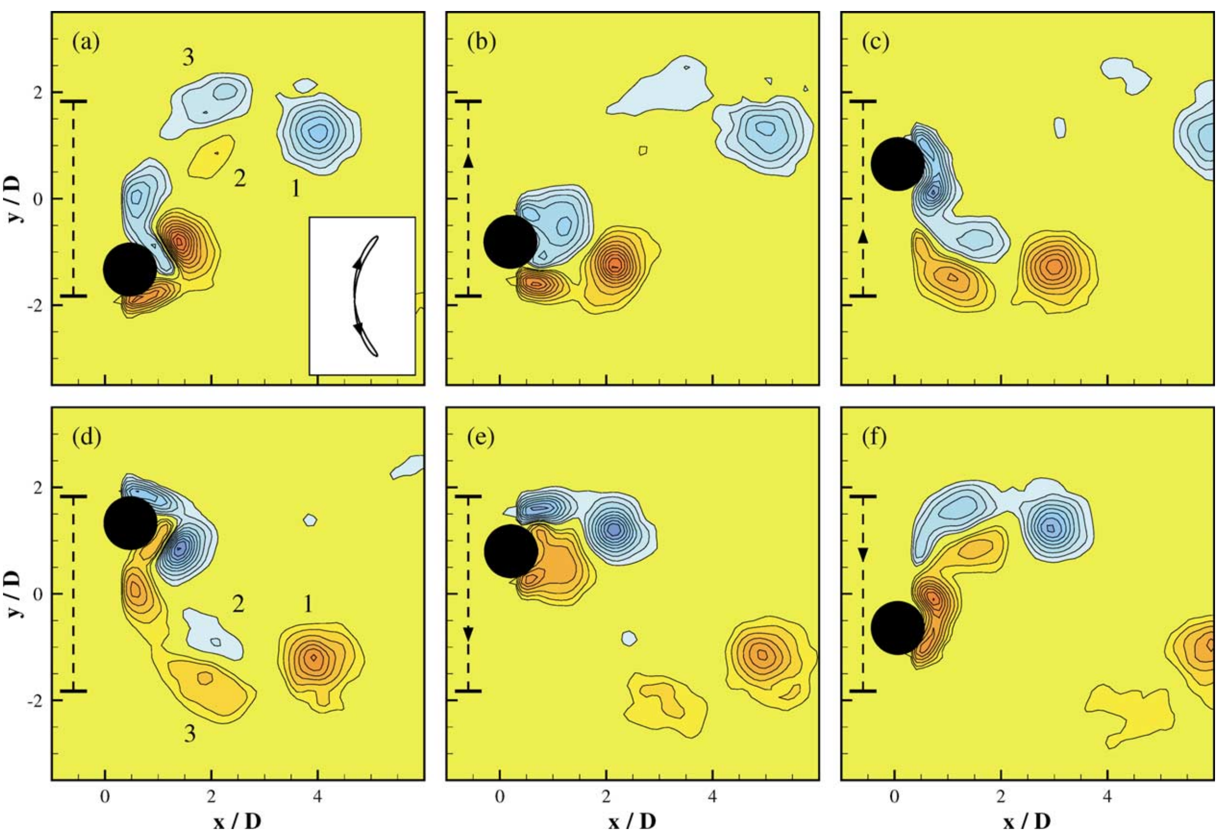


Fig. 4. "2T" vortex wake mode. This new mode of vortex formation comprises a triplet of vortices to form in each half cycle (see vortices 1-2-3 for example). In this case, $m^* = 2.6$; $Re = 5300$; $A_Y^* = 1.33$; $(U^*/f^*)S = 1.48$. Vorticity contours are $\{\omega D/U = \pm 0.4, \pm 1.2, \pm 2.0, \ldots\}$.

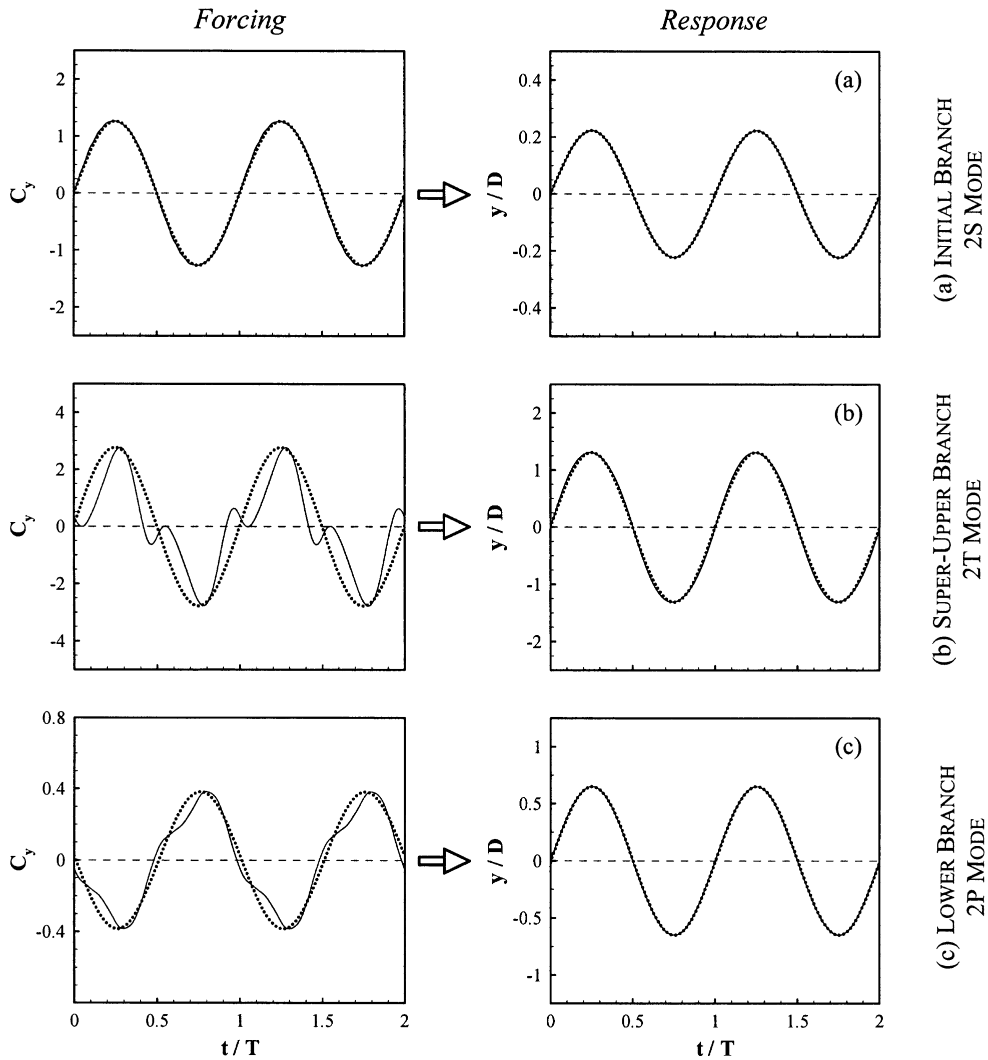


Fig. 5. Time traces of transverse force coefficient (C_Y) and response (y/D) for XY-motion. Average cycles are for the Initial Branch (a), Super–Upper Branch (b), and the Lower Branch (c). In each case, a pure sine wave (shown as dotted lines) with the same amplitude and frequency has been superimposed on the raw signal (solid lines). (Reduced velocities are $(U^*/f^*)S = 1.13$, 1.48, and 1.37 for the Initial, Super–Upper, and Lower Branches respectively.)

The major difference at this point in the cycle, for the 2T mode, appears to be the third principal vortex (labelled as 3) which is generated *in addition* to the classical vortex pair. Thus, we can see that the essential difference between the modes is the extra vortex 3, which we might expect is responsible for the markedly different system dynamics, as we show in detail, by considering energy transfer from the vortex motions to body motions, in Jauvtis and Williamson [19].

If we focus now on Fig. 3(e) and (f), it seems that it is the significant *acceleration* of the body at the top of the cycle that generates a fresh pair of vortices (vortices 2 and 3), which are placed in proximity to the strongest vortex 1. (Such a vortex 1 appears to form in all of the observed vortex modes, namely the 2T, 2P and 2S modes.) An illustration of this "starting" vortex pair (vortices 3 and 2), due to the strong acceleration, is particularly evident in frame f.

There has been some discussion recently concerning the validity of using sinusoidal forced vibrations to determine the induced forces, and thereby to predict what would happen for elastically-mounted structures. The present vortex dynamics for the 2T mode indeed yield a transverse force variation that is distinctly non-sinusoidal, as we see in Fig. 5(b), for the super-upper branch. However, such forces imposed on the elastically-mounted body induce a remarkably sinusoidal response. This interesting result is explained in detail in Williamson and Jauvtis [17], where even the overwhelmingly non-sinusoidal "vortex force" variations lead to markedly sinusoidal body response. With some care, the data from forced vibration studies will accurately yield predictions of free body response.

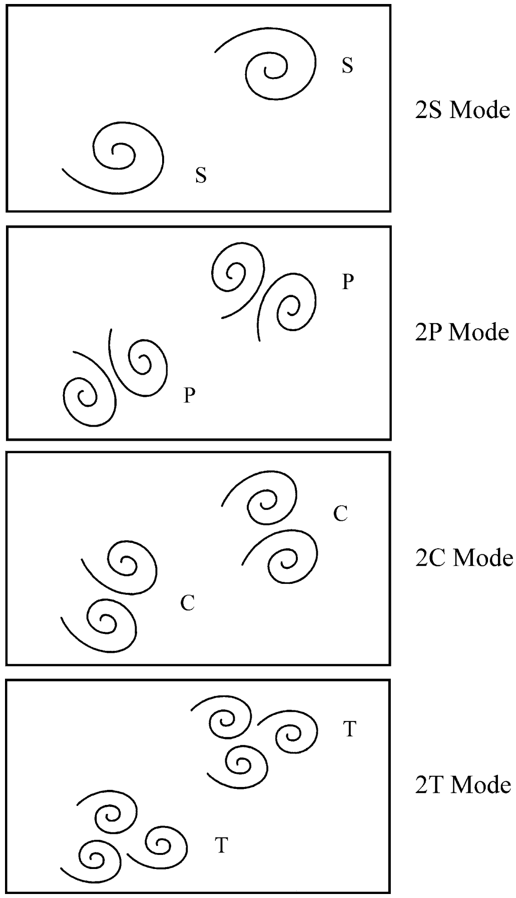


Fig. 6. The set of vortex wake modes {2S,2P,2C,2T} which are known to contribute to vortex-induced vibration of cylindrical structures. All of these modes are antisymmetric. The 2C mode is found for the vibration of a pivoted cylinder (Flemming and Williamson [20]). The 2T mode is found for XY motion. The 2S and 2P modes are observed in all these cases so far studied.

4. Concluding remarks

Despite the large number of papers dedicated to the problem of a cylinder vibrating transverse to a fluid flow (Y-motion), there are very few papers which allow the body to vibrate in-line with the flow, as well as transverse to the free stream. Surprisingly, there are no experiments (to our knowledge) which address the most practical problem, namely a body in two degrees of freedom (XY-motion) where the oscillating mass is precisely the same in the transverse and in-line directions, and where the natural frequency is the same in both directions. For this purpose, we have designed the present pendulum apparatus to achieve both of these criteria.

Even down to the low mass ratios, where $m^* = 6$, it is remarkable that the freedom to oscillate in-line with the flow, affects the transverse vibration surprisingly little. The same response branches, peak amplitudes, induced forces, and vortex wake modes are found for both Y-only and XY motion. There is, however, a dramatic change in the fluid-structure interactions when mass ratios are reduced below $m^* = 6$. A new response branch with significant streamwise motion appears, in what we call the “super-upper” branch, yielding massive amplitudes of 3 diameters peak-to-peak ($A_Y^* \sim 1.5$), which is far in excess of typical peak amplitudes for Y-only vibration ($A_Y^* \sim 1$). To present the response measurements, we introduce a three-dimensional plot where the Williamson–Roshko map of regimes (Y-only motion) forms the horizontal plane $\{(U^*/f^*)S$ versus $A_Y^*\}$, and where vertical height represents streamwise amplitude, A_X^*. The super-upper branch flies above the Williamson–Roshko map of regimes, but interestingly it both starts and terminates directly above the boundaries of the 2P mode for Y-only motion. Our super-upper branch is but one line within a possible continuous surface of solutions for XY free vibration, which may exist in the space above the horizontal Williamson–Roshko regime map, and for which there would be positive energy transfer from fluid to body motion.

The discovery of a super-upper response branch corresponds with a new periodic vortex wake mode. This mode comprises a triplet of vortices being formed in each half cycle, in what we define as a "2T" mode, following the terminology introduced in Williamson and Roshko [7]. The triplet strongly resembles the vortex pair formed in each half cycle in the 2P mode, with the addition of a third strong vortex formed during the acceleration phase near the extremities of the transverse motion. It appears that the 2T mode is the fourth vortex wake mode which contributes to vortex-induced vibration of such cylindrical structures. We summarise, in Fig. 6, the set of four vortex wake modes:

{2S,2P,2C,2T}

all of which have an antisymmetric symmetry. The 2C mode comprises a distinct pair of co-rotating vortices of the same sign generated in each half cycle, and is found in the pivoted cylinder experiments of Flemming and Williamson [20]. The 2T mode is found in the present XY motion experiments. The 2S and 2P modes are found in all cases including the transverse-only cylinder vibration. All of these vortex wake modes are generated by the body motion, while the vortices in turn provide positive energy transfer to continuously support the body motion.

In conclusion, we find that the freedom to move streamwise to the flow, as well as transverse, hardly changes the dynamics of elastically-mounted cylinders, in the case of moderate mass ratios, $m^* > 6$. This indicates that the phenomena, and the extensive understanding, of vortex-induced vibration for Y-only body motions, built up over the last 35 years, remain of strong relevance to the case of two degrees of freedom. It is only for "small" mass ratios, $m^* < 6$, that one observes a rather dramatic departure from previous results, yielding huge amplitudes of vibration, which would suggest a possible modification to the offshore design codes that have traditionally been based on results for bodies vibrating only transverse to the flow.

Acknowledgements

The support from the Ocean Engineering Division of the O.N.R., monitored by Tom Swean, is gratefully acknowledged. (O.N.R.Contract No. N00014-95-1-0332.) Many thanks are due to Matt Horowitz for his enthusiastic help in preparing these figures.

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